Competitive Evaluation of Failure Detection Algorithms for Strapdown Redundant Inertial Instruments

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Seven algorithms for failure detection, isolation, and correction of strapdown inertial instruments in the dodecahedron configuration are competitively evaluated by means of a digital computer simulation that provides them with identical inputs. Their performance is compared in terms of orientation errors and computer burden. The analytical foundations of the algorithms are presented. The features that are found to contribute to superior performance are use of a discrete logical structure, elimination of interaction between failures, different thresholds for first and second failures, use of the "parity" test signals, and avoidance of iteration loops.

Nomenclature

A = 6 × 3 matrix of input axis direction cosines
$A(k)$ = A with kth row set equal to zero
B = 3 × 6 solution matrix
C = 3 × 6 test signal matrix
c = $\cos\alpha$
D = 15 × 6 test signal matrix
d = $5^{-1/2}$
F = 15 × 3 test signal matrix
f = 3-vector of filtered inconsistency states
k_A = prefilter gain
k_B = prefilter gain
Q = 6 × 6 diagonal instrument error covariance matrix
$\hat{q}$ = 6-vector of unbiased variance estimates
s = $\sin\alpha$
u = 15-vector of indirect or parity test signals
v = 3-vector of inconsistency states
x = 3-vector of strapdown package angular velocity or acceleration
$\hat{x}$ = estimate of x
y = 6-vector of instrument outputs
$\hat{y}$ = estimate of what y would be in the absence of errors
α = half the angle between any pair of instrument input axes
ε = 6-vector of instrument errors
$\hat{\varepsilon}$ = estimate of ε or residuals

Introduction

THE idea of improving the reliability of a system by using redundant elements and some method of reorganization after a failure has been in existence for some time. Since inertial instruments have never been as reliable as one might like, inertial navigators and attitude references have been prime candidates for this treatment. System level redundancy has been most common, with duplicated or triplicated gimballed platforms or strapdown packages. Duplication permits automatic failure detection but requires some kind of external information for failure isolation. Triplication permits automatic failure detection and isolation by a simple majority voting scheme. However a second failure cannot always be isolated with triplicated systems.

Weiss and Nathan[1] seem to have been first to note that, if six inertial instruments are arranged so that no three of their input axes are coplanar, then first and second failures can be detected and isolated. (It appears to be most practicable to embody this concept in a strapdown package, thus avoiding the problem of providing gimbal redundancy. However, it is possible, for example, to put three of the instruments on each of two gimballed platforms and use data crossfeeding and gimbal slaving techniques to enhance reliability. Only the strapdown case is considered here.) Thus greater reliability can be achieved than with nine instruments arranged three per orthogonal axis. A third failure cannot be isolated without external information.

Ephgrave[2] and Gilmore[3] have shown that the optimal arrangement for the six instruments is with their input axes perpendicular to the faces of a regular dodecahedron. Both gyros and accelerometers can be arranged in this manner. The symmetry of the dodecahedron configuration maximizes accuracy for the worst cases of operation with a subset of the instruments.

The simple majority voting scheme is no longer usable because the instrument outputs are not directly comparable, since their input axes all point in different directions. Therefore, an algorithm must be devised to perform failure detection, isolation, and correction (FDIC).

Many such algorithms have been suggested by various authors. This paper compares seven FDIC algorithms with one another by simulation to determine which is the best and to obtain insight into their modes of operation with the goal of combining their best features and remedying their shortcomings.[4]

The problem has two aspects. The first is the problem of detecting signals (the errors of the failed instruments) in the presence of noise (the errors of the unfailed instruments) and making a decision as to the existence of one or more failures. The second aspect is the problem of isolating the failed instruments. This problem is not trivial, because the available information can be ambiguous in the case of two failures. Of course, simultaneous failures should be quite unlikely in a well-designed system, but one should not overlook the possibility that a second failure could occur during the finite time required to detect and isolate a first failure. It is also conceivable that a first failure might be small enough to go undetected and yet be able to interfere with the detection of a subsequent failure because of the ambiguity mentioned above.

Analytical Foundation of the Algorithms

The instrument outputs are

$$y = Ax + \varepsilon \tag{1}$$

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where y = 6-vector of instrument angular velocity or acceleration outputs; $A = 6 \times 3$ matrix of instrument input axis direction cosines with respect to axes fixed in the strapdown package; x = 3-vector of instrument package angular velocity or acceleration; and ε = 6-vector of instrument errors.

For all of the algorithms, we take

$$A = \begin{bmatrix} c & s & 0 \\ c & -s & 0 \\ 0 & c & s \\ 0 & c & -s \\ s & 0 & c \\ -s & 0 & c \end{bmatrix} \tag{2}$$

where

$$c = \cos\alpha = [0.5 + (0.05)^{1/2}]^{1/2} \tag{3}$$

$$s = \sin\alpha = [0.5 - (0.05)^{1/2}]^{1/2} \tag{4}$$

where α is half the angle whose tangent is 2. The ratio s/c is equal to the golden mean. The rows of A are unit vectors perpendicular to the faces of a dodecahedron. The columns of A are orthogonal 6-vectors of length $2^{1/2}$. It is possible to choose three more 6-vectors of length $2^{1/2}$ which are orthogonal to the columns of A and to each other. Let them comprise the rows of a matrix C. One of the infinite number of possibilities is

$$C = \begin{bmatrix} s & s & 0 & 0 & -c & c \\ -c & c & s & s & 0 & 0 \\ 0 & 0 & -c & c & s & s \end{bmatrix} \tag{5}$$

By definition

$$CA = 0 \tag{6}$$

Let v be a 3-vector of inconsistency states, where

$$v = Cy \tag{7}$$

From Eqs. (1, 6, and 7)

$$v = C\varepsilon \tag{8}$$

Thus the inconsistency states are independent of the strapdown package intput angular velocity or acceleration and depend only on the instrument errors. All of the test signals used in the various algorithms can be expressed as linear or quadratic combinations of the components of v.

Most of the algorithms have a prefilter which reduces the effect of high-frequency noise, such as quantization error, on performance. The prefilter is a first-order filter such as

$$f^{(i+1)} = k_A f^{(i)} + k_B v^{(i+1)} \tag{9}$$

where

$$k_A = \exp(-T/\tau_f) \tag{10}$$

$$k_B = 1 - k_A \tag{11}$$

with T being the sampling period and τ_f the filter time constant.

When the failures have been detected and isolated, it is necessary to correct for them. In two algorithms correction is accomplished by obtaining the weighted least-squares estimate of x.

$$\hat{x} = By \tag{12}$$

$$B = (A^T Q^{-1} A)^{-1} A^T Q^{-1} \tag{13}$$

where Q is the covariance matrix of the instrument errors and is assumed to be diagonal. In most of the other algorithms the failed instruments are excluded from use so that Q_{ii}^{-1} is 0 for a failed and 1 for an unfailed instrument. Thus Eq. (13) becomes the least-squares estimate

$$B = (A^T A)^{-1} A^T \tag{14}$$

where the rows of A corresponding to zero Q_{ii}^{-1} are set equal to zero.

Several of the algorithms make use of the residuals of the least-squares solution as "direct" test signals, using them as estimates of the instrument errors. They are given by

$$\hat{\varepsilon} = y - \hat{y} \tag{15}$$

where

$$\hat{y} = A\hat{x} \tag{16}$$

so that, from Eqs. (12, 15, and 16)

$$\hat{\varepsilon} = (I - AB)y \tag{17}$$

For the case of no failures, Q is the identity matrix and

$$B = \tfrac{1}{2} A^T \tag{18}$$

One can show by direct calculation that

$$I - \tfrac{1}{2} AA^T = \tfrac{1}{2} C^T C \tag{19}$$

so that the $\hat{\varepsilon}_i$ are linear combinations of the v_j

$$\hat{\varepsilon} = \tfrac{1}{2} C^T C y = \tfrac{1}{2} C^T v \tag{20}$$

Thus the six residuals contain no more information than the three inconsistency states. Prefiltered values of $\hat{\varepsilon}$ can be obtained from

$$\hat{\varepsilon} = \tfrac{1}{2} C^T f \tag{21}$$

where f is given in Eq. (9). This formulation requires the prefiltering of only three quantities, rather than six.

When the kth instrument has failed, the residuals are given by Eqs. (14) and (17) with the kth row of A set equal to zero to give $A(k)$.

$$\hat{\varepsilon} = \{I - A(k)[A(k)^T A(k)]^{-1} A(k)^T\} y \tag{22}$$

One can show by direct calculation that Eq. (22) is equivalent to

$$\hat{\varepsilon} = \tfrac{1}{2} C(k)^T C y = \tfrac{1}{2} C(k)^T v \tag{23}$$

where

$$C_{ij}(k) = C_{ij} - \left(\sum_{m=1}^{3} C_{mj} C_{mk} \right) C_{ik} \tag{24}$$

with the exception that $\hat{\varepsilon}_k$, the residual of the failed instrument, is given as

$$\hat{\varepsilon}_k = y_k \tag{25}$$

by Eq. (22) and as

$$\hat{\varepsilon}_k = 0 \tag{26}$$

by Eq. (23). The latter is better, since the residual of the failed instrument has to be ignored, and Eq. (26) causes it to be ignored automatically.

For the case of zero failures, the residuals may be calculated from Eqs. (1, 5, 6, and 20) to give

$$\hat{\varepsilon} = \tfrac{1}{2} \begin{bmatrix} 1 & -d & -d & -d & -d & d \\ -d & 1 & d & d & -d & d \\ -d & d & 1 & -d & -d & -d \\ -d & d & -d & 1 & d & d \\ -d & -d & -d & d & 1 & -d \\ d & d & -d & d & -d & 1 \end{bmatrix} \varepsilon \tag{27}$$

where

$$d = 5^{-1/2} \tag{28}$$

The residuals may be considered to be estimates of the instrument error values. Their squares may be considered to be estimates of the variances of the instrument errors, but they are biased estimates. One can show that unbiased estimates of the instrument error variances are given by

$$\hat{q}_i = 5\hat{\varepsilon}_i^2 - \frac{1}{2} \sum_{j=1}^{6} \hat{\varepsilon}_j^2 \tag{29}$$

We now consider the "indirect" or "parity" test signals which, instead of trying to put in evidence the error of a particular instrument, show the error of a group of instruments that excludes two particular instruments. There are fifteen such signals (six instruments taken two at a time). Requiring that the signals be independent of the input angular velocity or translational acceleration determines them to within a constant factor. If u is the 15-vector of test signals, then

$$u = Dy \tag{30}$$

where a typical D matrix is

$$D = \begin{bmatrix} -c & c & s & s & 0 & 0 \\ s & -c & -c & 0 & s & 0 \\ c & -s & -c & 0 & 0 & s \\ -c & s & 0 & c & s & 0 \\ -s & c & 0 & c & 0 & s \\ s & s & 0 & 0 & -c & c \\ -s & 0 & -s & c & c & 0 \\ s & 0 & -c & s & 0 & c \\ c & 0 & -s & 0 & -s & c \\ c & 0 & 0 & -s & -c & s \\ 0 & -s & -c & s & c & 0 \\ 0 & s & -s & c & 0 & c \\ 0 & c & s & 0 & -c & s \\ 0 & c & 0 & s & -s & c \\ 0 & 0 & -c & c & s & s \end{bmatrix} \tag{31}$$

Each component of u is a linear combination of the components of the inconsistency state vector v. Thus the matrix D can be factored into two matrices

$$D = FC \tag{32}$$

and

$$u = FCy = Fv \tag{33}$$

Prefiltering may be performed on the three components of v rather than the 15 components of u. Thus we replace Eq. (33) by

$$u = Ff \tag{34}$$

where f is given by Eq. (9).

Algorithms

Eight different algorithms were selected for competitive evaluation but only seven were simulated. In roughly chronological order they are 1) Adaptive 66,[5] 2) Fifteen Threshold,[6] 3) Squared Error,[7] 4) Bayesian Decision Theory,[8] 5) Maximum Likelihood,[9] 6) Minimax,[10] 7) Adaptive 72 (not simulated),[11] and 8) Sequential.[12] The algorithms are described briefly here and in more detail in Ref. 4. However, the reader must refer to Refs. 5–12 for the theoretical backgrounds and derivations of the algorithms which, in many cases, are quite lengthy.

The Adaptive 66 Algorithm uses a weighted least-squares estimator to perform failure detection, isolation, and correction. The estimator estimates the strapdown package input vector (acceleration or angular velocity). The residuals are squared to obtain estimates of the instrument error variances which are used to get the weights of the estimator. New estimates are then obtained iteratively until the variances stop changing. A prefilter, which filters the six instrument outputs prior to their use in the algorithm, has been added to allow it to compete on an equal basis with the other algorithms.

The Maximum Likelihood Algorithm is very similar, but uses the six unbiased variance estimates of Eq. (29) to start the iteration. The inconsistency vector is modified by subtraction of the residuals to reduce the interaction of successive failures. The squared residuals are added to the variance estimates to compensate for the decrease in the unbiased variance estimates that would otherwise result from the subtraction of the residuals. The algorithm has a prefilter like that of the Adaptive 66 Algorithm.

The Bayesian Decision Theory Algorithm performs failure correction by means of a least-squares estimator using only the instruments classified as unfailed. To detect and isolate the first failure, the algorithm compares the largest residual from Eq. (20) with a threshold. If the threshold is exceeded, the corresponding instrument is failed. To detect and isolate the second failure, Eq. (23) and a different threshold level are used. A prefilter has been added to the algorithm to allow it to compete on an equal basis with the other algorithms.

The Squared Error Algorithm performs failure correction by means of a least-squares estimator like that of the Bayesian Decision Theory Algorithm. To detect the first failure, the sum of the squares of the residuals (total squared error) from Eq. (20) is compared with a threshold. If the threshold is exceeded, isolation is accomplished by comparing the ratios of the squared residuals to the total squared error with a different threshold. Detection and isolation of the second failure is similar, using Eq. (23) and two additional thresholds. The prefilter of the algorithm has been replaced by the prefilter of Eq. (9) to prevent prefilter differences from obscuring the relative performance of the algorithms themselves.

The Fifteen Threshold Algorithm performs failure correction by means of a least-squares algorithm like those of the preceding two algorithms. Failure detection and isolation are performed using the fifteen parity test signals and a single threshold level. Whenever a test signal exceeds the threshold, it is "locked out" so that it cannot recover later on. If one or more of the ten test signals to which the output of a given instrument contributes has never exceeded its threshold, that instrument is considered unfailed. Otherwise it is considered failed. The algorithm has a prefilter like that of the preceding two algorithms.

The Minimax Algorithm offers two different methods of failure correction. One is a least-squares estimator like those of the preceding three algorithms. The other is the Bounding Sphere Algorithm which minimizes the maximum possible estimation error. Failure detection and isolation are performed using the fifteen parity test signals and a single threshold level. For the first failure, detection occurs when any signal exceeds the threshold. For isolation of the first failure, it is assumed that only one failure can occur at a time. The four instruments that contribute to the value of a particular parity signal are called a quartet. If the signal exceeds its threshold, the quartet is dirty; if not, it is clean. With six instruments and one failure, there must be one set of five good instruments. With a set of five instruments called a quintet, the isolation algorithm finds every quintet whose quartets are all clean. Such a quintet may be a set of good instruments and will be called a clean quintet. An instrument which is excluded from every clean quintet must be failed since a good instrument would be in some clean quintet. Thus the failed instrument is the one excluded from all clean quintets. For the second failure, detection occurs when any of the five parity signals which are independent of the output of the first failed instrument exceeds the threshold. When four of the five signals exceed the threshold, the second failure is successfully isolated. The zeros in the equation for the sole clean parity signal correspond to the two failed instruments. Although the threshold is the same for first and second failures, its "effective" level is 4.2 db higher for second failures.[4] A prefilter has been added to the algorithm to allow it to compete on an equal basis with the other algorithms.

The Sequential Algorithm uses a Kalman-Bucy filter to perform failure correction. The filter has nine states: three for the package input vector and six for the instrument errors, all modeled as first-order Gauss-Markov processes. The measurements are the six instrument outputs. Two different types of threshold are used. The first type is constant, is set quite high, and is applied to the Kalman-Bucy residuals. If the threshold is exceeded, the associated instrument is excluded from the measurement vector. When the residual is again less than the threshold, the instrument is reinstated. The second type is proportional to the standard deviation of the error state as given by the filter covariance matrix, is set much lower, and is applied to the instrument error states. An instrument whose error state exceeds the threshold by more than any other error state is permanently excluded from the measurement vector. No prefilter is required. One can show that the Kalman-Bucy filter, in the steady state, approximates the prefilter of Eq. (9) and that the instrument error states approximate the filtered residuals.[4]

The Adaptive 72 Algorithm had not been documented in sufficient detail in time for its inclusion in the simulation. Its most interesting feature is its filter (or "detection system") which

Table 1 Algorithm comparison

Algorithm	Test signals for first failure	Test signals for second failure	Failure correction
Adaptive 66	6 Residuals	6 Residuals	Weighted least squares
Fifteen Threshold	15 parity signals	5 parity signals	Least squares
Squared Error	6 squared residuals and their sum	5 squared residuals and their sum	Least squares
Bayesian Decision Theory	6 residuals	5 residuals	Least squares
Maximum Likelihood	6 variance estimates and 6 residuals	6 variance estimates and 6 residuals	Weighted least squares
Minimax	15 parity signals	5 parity signals	Least squares or bounding sphere
Adaptive 72	6 parity signals	5 parity signals	Least squares
Sequential	6 filter states	5 filter states	Kalman-Bucy filter

is nonlinear. It also identifies ramp and bias type errors, performs recompensation, and reinstates instruments. Table 1 compares the principal characteristics of the algorithms.

The Simulation

The purpose of the simulation program is to provide up to eight algorithms with identical environments which are more complex and more realistic than the models used in their derivations. The trajectory generator provides an ideal trajectory (a 400 sec Shuttle boost) that includes rotational and translational vibratory motions and staging transients. Ideal attitude and inertial velocity and the components of acceleration and angular velocity along all three axes of each instrument are calculated. The errors of the instruments in unfailed operation are calculated including bias, random, scale factor, misalignments, quantization, and, for the gyros only, mass unbalance and anisoelastic drifts. The failure induced errors are bias, ramp, random, scale factor, zero output, maximum output, and, for the gyros only, mass unbalance. The different FDIC algorithms each combine the six gyro and six accelerometer outputs into incremental angle and velocity 3-vectors. FDIC algorithm 0 is a nominal algorithm supplied with information from the failure simulator so that it uses only unfailed instruments, providing a standard

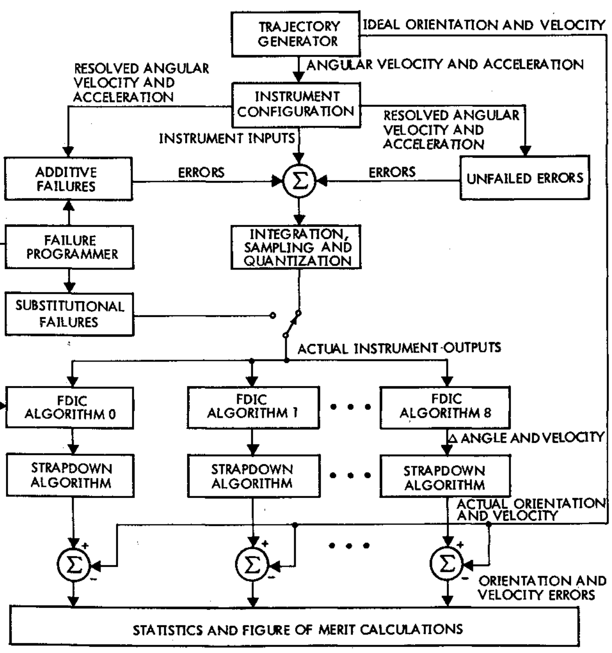

Fig. 1 Simulation over-all block diagram.

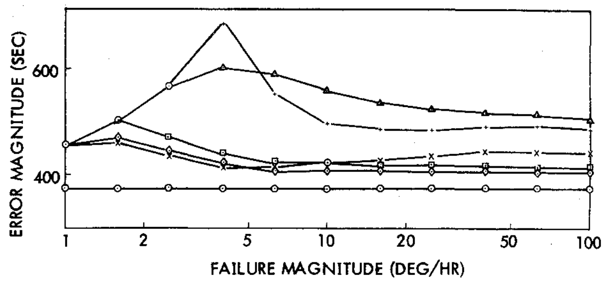

Fig. 2 Orientation error vs failure magnitude, first failure, $\tau = 100$ sec.

against which the real algorithms can be compared. Identical strapdown algorithms convert the incremental angles and incremental velocities into actual orientation and actual inertial velocity which are compared to the ideal values from the trajectory generator and the nominal values from the nominal algorithm to determine the errors. Missed and false alarm statistics and computer elapsed time are also obtained. Figure 1 shows the simulation over-all block diagram. Although only the one environment is provided, and the absolute performance of the algorithms will vary in other environments, it is not anticipated that their relative performance will vary significantly over the range of environments appropriate to a system of this type.

Results

Before the results could be obtained, it was necessary to set all of the threshold levels and prefilter time constants for the different algorithms. Ideally, the parameters should be set optimally for each algorithm. To do so would have been quite difficult. Even just reaching an acceptable definition of optimality is a difficult task. It was decided that a reasonable approach, capable of being achieved in a limited number of computer runs, would be to set all of the thresholds for the same false alarm rate. Thus all algorithms would give the same performance in the absence of failures (the most probable situation for any particular mission) and their relative performance in the presence of failures would establish their order of merit.

Two values of the time constants were chosen arbitrarily, 10 and 100 sec. The thresholds were set by increasing them in 2 db steps until no false alarms were experienced in the cases of zero, one, and two gyro bias shift failures. (Unfortunately, it was not possible to complete parameter setting for the accelerometers.) It was found that higher threshold levels were necessary for one failure than for zero failures, and that higher levels were necessary for two failures than for one failure. Thus the first failure performance was degraded for those algorithms which did not have separate thresholds for second failures, except for the Minimax Algorithm which has an "effective" threshold level higher for

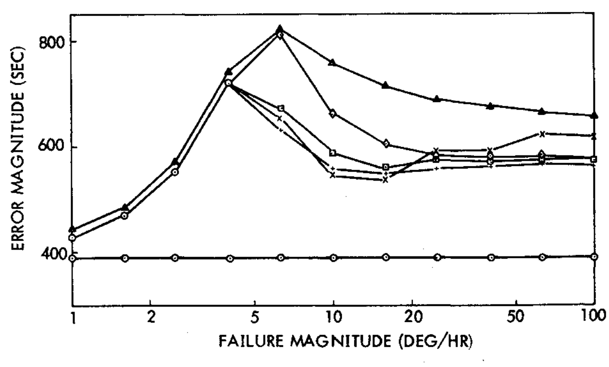

Fig. 3 Orientation error vs failure magnitude, second failure, $\tau = 100$ sec.

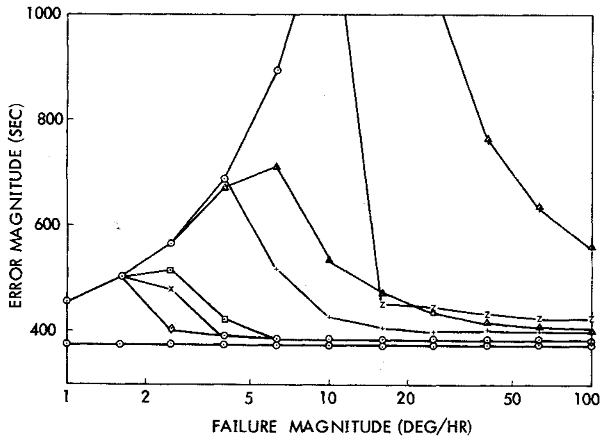

Fig. 4 Orientation error vs failure magnitude, first failure, $\tau = 10$ sec.

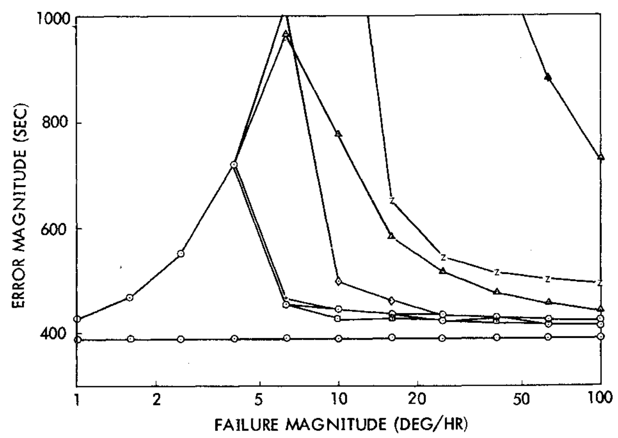

Fig. 5 Orientation error vs failure magnitude, second failure, $\tau = 10$ sec.

second failures than for first failures. For the 100 sec time constant prefilter, it was not possible to find threshold settings for the second failures that eliminated false alarms without eliminating true alarms as well for the Sequential and Maximum Likelihood Algorithms.

A series of step bias failures of varying magnitudes were simulated at 200 sec. Figures 2 and 3 show the orientation error with respect to the ideal attitude at the end of boost vs failure magnitude for the first and second failures for the 100 sec time constant and Figs. 4 and 5 are for the 10 sec time constant. Table 2 identifies the symbols for the different algorithms.

Table 2 Symbols

Symbol	Algorithm
Δ	Adaptive 66
+	Fifteen Threshold
×	Squared Error
◇	Bayesian Decision Theory
⍋	Maximum Likelihood
□	Minimax
Z	Sequential
○	Nominal

A series of 21 cases with double simultaneous failures of different types was run as a severe test of the capabilities of the four algorithms that performed best in the previous cases. The 10 sec time constant was used. The orientation error with respect to the ideal attitude and with respect to the nominal algorithm attitude was averaged over the 21 cases with the results shown in Table 3.

Table 3 Double simultaneous failures

Algorithm	Ideal (arcsec)	Nominal (arcsec)
Nominal	434	0
Fifteen Threshold	688	396
Squared Error	761	465
Bayesian Decision Theory	841	563
Minimax/Least Squares	672	366

By combining the abovementioned in a somewhat subjective manner, the following ordering of the algorithms by decreasing performance can be obtained: Minimax, Squared Error, Bayesian Decision Theory, Fifteen Threshold, Adaptive 66, Sequential, and Maximum Likelihood.

The computer burden of the algorithms may be compared by examining the CDC 6400 computer words and central processor time required for a typical 400 sec case. The sampling period was 125 msec. The computer burden is shown in Table 4. However, because of their iterative nature, the Adaptive 66 and Maximum Likelihood Algorithms, on occasion, can consume much more time, even more than 400 sec.

Table 4 Computer Burden

Algorithm	Size (words)	Time (sec)
Adaptive 66	616	10.6
Fifteen Threshold	478	8.5
Squared Error	761	6.3
Bayesian Decision Theory	663	7.0
Maximum Likelihood	842	15.6
Minimax	867	14.6
Sequential	807	188.0

Conclusions

A good FDIC algorithm has a discrete logical structure that makes explicit what instruments are classified as failed or unfailed, what hypotheses are being tested, and what test signals and thresholds are to be used in testing each different hypothesis.

Interaction between failures should be eliminated as much as possible by never using the output of an instrument which has been classified as failed in making a decision about the classification of another instrument.

Independently adjustable thresholds for first and second failures or a properly preset ratio between the thresholds for first and second failures should be provided.

The indirect (parity) test signals seem to be better than the direct (residual) test signals.

Iterative algorithms should be avoided.

The choice of filter time constant (degree of reliance upon past data history) significantly affects the shape of the error magnitude vs failure magnitude curves.

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Selected Applications of a Biaxial Tiltmeter in the Ground Motion Environment

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The features of the biaxial tiltmeter that allow its utilization for monitoring various seismic phenomena are discussed. Long-term earthtide level data are presented comparing the sensitivity threshold and stability with laboratory standard mercury tube tiltmeters. Use of the device for earthquake prediction capability is explored through presentation of data showing precursor tilts for a large number of seismic events.

Introduction

DESIGN characteristics and applications of the two-axis electrolytic bubble level as a primary vertical reference in missile systems have been presented in a previous paper.[1] Recent major innovations in fabrication techniques made with respect to the tilt sensor have a strong bearing upon its integrity as a standard instrument for measuring seismic phenomena. It is deemed appropriate, therefore, to mention some of the innovations and elaborate a little more fully on data that can provide insight into the capability of the instrument. When measuring data in the earthtide amplitude range (1 μrad), it is necessary for one to have a comprehensive understanding of the characteristics of the instruments to be utilized as well as some feeling for the environment in which the instruments are to perform. Clearly, the problem of defining instrument drift characteristics on a time scale consistent with the requirements of "earthtide" level measurements poses a great difficulty. If a high confidence level can be established in the drift and reliability characteristics of the instrument itself based upon sound prior investigation, then the acceptability of the measurement data will be maintained, though it may not conform to an expected behavioral pattern. It is recognized that data from multiple instruments located in close proximity to each other often provides a means for separating instrument error from the effects being measured, but cost considerations do not often allow the luxury of having the described redundancy feature, especially at remote seismic field installations such as utilized in earthquake prediction studies and volcano activity studies. In such studies, the inherent cost for a large number of instruments to map the area of interest precludes the use of more than one instrument in each site except in unusual circumstances. Thus, it becomes worthwhile for considerable effort to be devoted toward proving the instrument prior to its acceptance as a dependable device for use in the various projects where tiltmeter data are deemed useful. Because the sensing element of the tiltmeter was developed initially for military purposes, the focus of this development effort was upon long-term reliability. Thus, in shifting the application for the device over to a nonmilitary role, the reliability feature was a built-in bonus for these other applications. The role of proving the tiltmeter for use and acceptance as an instrument for eathquake fault monitoring and volcano erruption studies has been left to the using agencies such as the U.S. Geological Survey and California Division of Mines and Geology since they are the primary commercial customers to first utilize the device in this nonmilitary role. Discussions to follow will cover techniques and data utilized to prove the instrument and to gain a knowledge

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Index categories: LV/M Guidance Systems; Research Facilities and Instrumentation.

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